# Adiabatic loading of a Bose-Einstein condensate in a 3D optical lattice

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We experimentally investigate the adiabatic loading of a Bose-Einstein condensate into an optical lattice potential. The generation of excitations during the ramp is detected by a corresponding decrease in the visibility of the interference pattern observed after free expansion of the cloud. We focus on the superfluid regime, where we show that the limiting time scale is related to the redistribution of atoms across the lattice by single-particle tunneling.

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The observation of the superfluid to Mott insulator (MI) transition [1] undergone by an ultracold Bose gas in an optical lattice has triggered a lot of experimental and theoretical activities (see [2, 3, 4]). This system allows to experimentally produce strongly-correlated quantum systems in a well controlled environment, with applications ranging from the realization of novel quantum phases in multi-component systems (see, e.g., [4] and references therein) to the implementation of collisional quantum gates [5, 6, 7].

Most of these proposals rely on producing a system very close to its ground state. However, in experiments so far, the successful production of an ultracold gas in the optical lattice relies on adiabatic transfer. The condensate is first produced and evaporatively cooled in order to minimize the thermal fraction. The almost pure condensate is subsequently transferred into the lattice potential by ramping up the laser intensities as slow as possible in order to approach the adiabatic limit. For too fast a ramp, excitations are generated in the system, which eventually results in heating after the cloud has equilibrated at the final lattice depth.

It is important in this respect to note the existence of two energy scales in this problem, related to the single-particle band structure on the one hand or to the many-body physics within the lowest Bloch band on the other hand. Adiabaticity with respect to the band structure is associated with the absence of interband transitions. The characteristic time scales are on the order of the inverse recoil frequency, typically hundreds of microseconds. Experimentally, adiabaticity with respect to the band structure is easily ensured, and can be checked by detecting atoms in the higher Bloch bands [8, 9]. Adiabaticity with respect to the many-body dynamics of the system involves considerably longer time scales (on the order of tens or even hundreds of milliseconds). Theoretical studies of the loading dynamics have been reported in [10, 11, 12, 13, 14, 15]. It is clear that non-adiabatic effects are more pronounced in the superfluid phase. The insulator phase is indeed expected to be quite insensitive to such effects, due to the presence of an energy gap. In this paper, we focus on the superfluid phase. Our goal is to to clarify experimentally how slowly the loading has to proceed to minimize unwanted excitations and heating.

To this aim, we make use of long range phase properties characteristic to a BEC. A key observable for ultracold Bose gases in optical lattices is the interference pattern observed after releasing the gas from the lattice and letting it expand for a certain time of flight [1, 8, 16, 17, 18, 19]. The contrast of this interference pattern is close to unity when most atoms belong to the condensate, but diminishes with the condensed fraction as the cloud temperature increases. Hence, the visibility of this interference pattern can be used to investigate the dynamical loading of a condensate into the optical lattice. We focus here on a specific lattice depth in the superfluid regime. We find a characteristic time scale of $\sim 100\,\mathrm{ms}$ above which the visibility of the interference pattern appears to be stationary. We show how it relates to the redistribution of atoms in the lattice through single-particle tunneling.

In our experiment, a $^{87}$Rb Bose-Einstein condensate is loaded into an optical lattice created by three orthogonal pairs of counter-propagating laser beams (see [1] for more details). The superposition of the lattice beams, derived from a common source at a wavelength $\lambda_L = 850$ nm, results in a simple cubic periodic potential with a lattice spacing $d = \lambda_L/2 = 425$ nm. The lattice depth $V_0$ is controlled by the laser intensities, and is measured here in units of the single-photon recoil energy, $E_r = h^2/2m\lambda_L^2 \approx h \times 3.2$ kHz. The optical lattice is ramped up in a time $\tau_{\mathrm{ramp}}$, using a smooth waveform that minimizes sudden changes at both ends of the ramp. The ramp form is calculated numerically. The program imposes that the lattice depth is initially zero and reaches its final value after a time $\tau_{\mathrm{ramp}}$. In between (for times $0 \le t \le \tau_{\mathrm{ramp}}$), it tries to match the functional form

$$V_0(t) = \frac{V_{\max}}{1+\exp\left(-\alpha \frac{t}{\tau_{\mathrm{ramp}}}\right)}, \qquad (1)$$

with $\alpha = 20$, by a piecewise linear approximation. The function (1) has been proposed in [12] and is shown in Fig. 1a. After this ramp, the cloud is held in the lattice potential for a variable hold time $t_{\mathrm{hold}}$ (see Fig. 1a), during which the system can re-thermalize. After switching

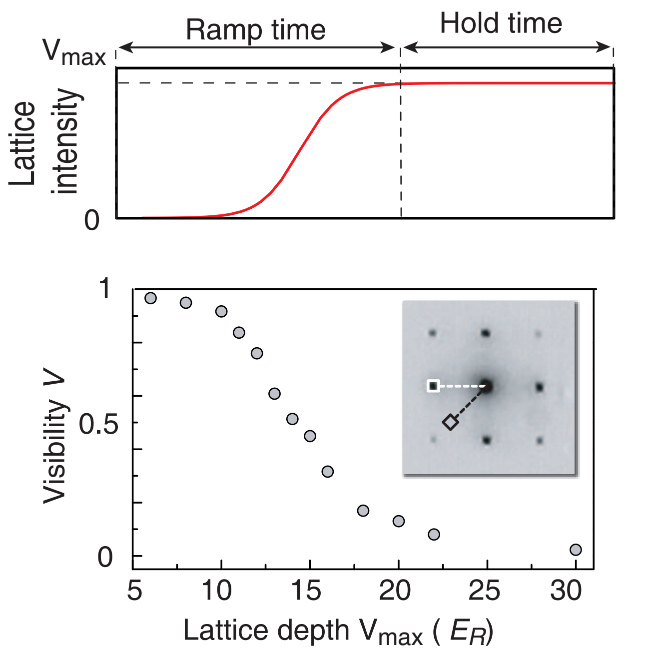


FIG. 1: (**a**)Sketch of the time profile used to ramp up the lattice depth to its maximum value $V_{\max}$. (**b**) Evolution of the visibility of the interference pattern as the lattice depth is increased. The set of data shown corresponds to $\sim 3 \times 10^5$ atoms (grey circles).

off the optical and magnetic potentials simultaneously and allowing for typically $t = 10 - 22$ ms of free expansion, standard absorption imaging of the atom cloud yields a two-dimensional map of the density distribution $n$ (integrated along the probe line of sight).

To extract quantitative information from time-of-flight pictures, we use the usual definition of the visibility of interference fringes,

$$\mathcal{V} = \frac{n_{\max} - n_{\min}}{n_{\max} + n_{\min}}. \tag{2}$$

In this work, we follow the method introduced in [18] and measure the maximum density $n_{\max}$ at the first lateral peaks of the interference pattern, (*i.e.* at the center of the second Brillouin zone). The minimum density $n_{\min}$ is measured along a diagonal with the same distance from the central peak (see inset in Fig. 1b). In this way, the Wannier envelope is the same for each term and cancels out in the division (see [18]). For a given two-dimensional absorption image, four such pairs exist and the corresponding values are averaged to yield the visibility. In addition, it is worth pointing out that this is an essentially model-independent characterization of the many-body system.

For comparison, we reproduce here measurements of the visibility as a function of lattice depth, similar to those presented in [18]. The data corresponds to a given total atom number $N \approx 3 \times 10^5$ atoms (grey circles in Fig. 1b). For lattice depths larger than 12.5 $E_r$, the system is in the insulating phase, as demonstrated in [1]. Yet, the visibility remains finite well above this point. For example, at a lattice depth of 15 $E_r$, the contrast is still around 30%, reducing to a few percent level only for a rather high lattice depth of 30 $E_r$. In [18], we have shown that such a slow loss in visibility is expected in fact even in the ground state of the system, being a consequence of the admixture of particle/hole pairs to the ground state for finite lattice depths. For these experiments, a ramp time of 160 ms followed by a hold time of 40 ms were used. We will show later on that this is slow enough to ensure adiabaticity.

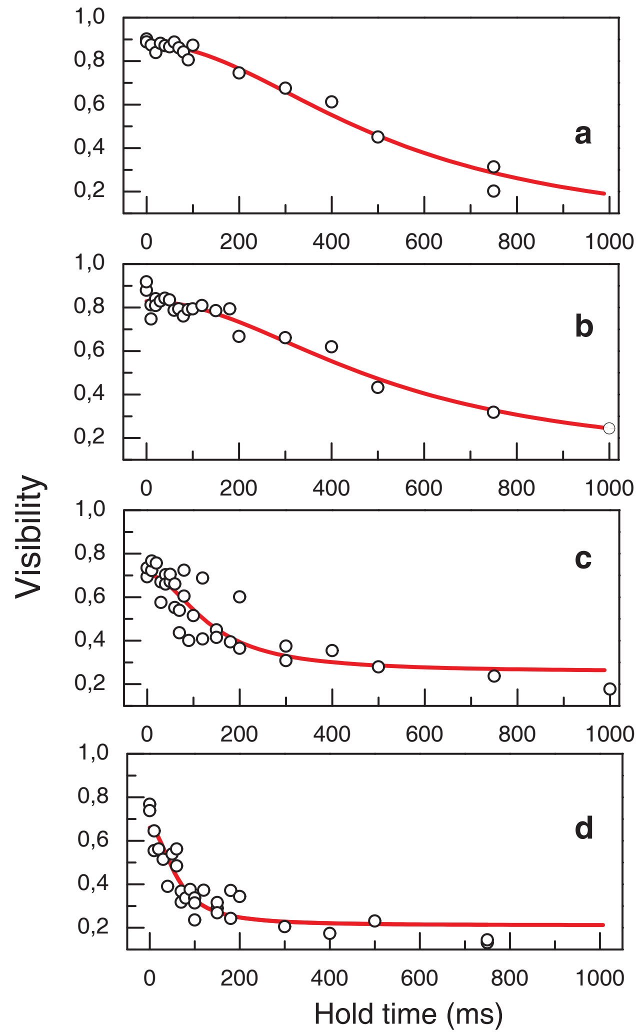


FIG. 2: Time evolution of the visibility for a fixed final depth $V_{\max} = 10\ E_r$ and various ramp times: (**a**): $\tau_{\mathrm{ramp}} = 160$ ms, (**b**): $\tau_{\mathrm{ramp}} = 80$ ms, (**c**): $\tau_{\mathrm{ramp}} = 40$ ms, and (**d**): $\tau_{\mathrm{ramp}} = 20$ ms.

We now focus on the superfluid regime, at a lattice depth $V_0 = 10\ E_r$. To investigate how the system is af-

fected by the ramping procedure, we fix the ramp time $\tau_{\rm ramp}$ and record how the interference pattern evolves as a function of hold time $\tau_{\rm hold}$. Examples of such measurements are shown in Fig. 2a-d.

For the slowest ramp shown, $\tau_{\rm ramp} = 160\,$ms, the visibility decreases with a time constant $\sim 500\,$ms. In fact, a very similar behavior is observed as soon as the ramp time exceeds $\tau_{\rm ramp} > 100\,$ms. Since this behavior is essentially independent of the ramp speed, it points to the presence of heating mechanisms which significantly degrade the visibility on a time scale of several hundred ms. The source of heating could be technical, due for instance to intensity or pointing fluctuations of the lattice beams, or intrinsic processes. An unavoidable process is for instance atomic spontaneous emission following excitations by one of the lattice beams. Here we attempt a crude estimate of the effect of spontaneous emission on visibility as follows. The heating rate $\Gamma_{\rm heat} \sim \Gamma_{\rm sp} \times (h^2/2m\lambda_0^2)$ is given by the rate $\Gamma_{\rm sp}$ at which such events happen due to *all* lattice beams, times the recoil energy $h^2/2m\lambda_0^2$, where $\lambda_0 \approx 780\,$nm is the resonant wavelength of the $D_2$ transition. To estimate the time scale $T_{\mathcal{V}}$ over which the visibility vanishes, we compute the time over which the energy gain per particle, $\Gamma_{\rm heat}T_{\mathcal{V}}$, is on the order of the critical temperature $T_c$ times Boltzmann's constant $k_B$. In a lattice potential with approximately one atom per site, we have $k_BT_c \sim zJ$, where $J$ is the tunneling matrix element and $z = 6$ the number of nearest neighbors in a three dimensional cubic lattice. This yields a final estimate

$$T_{\mathcal{V}} \sim \Gamma_{\rm sp}^{-1}\frac{zJ}{E_r}. \tag{3}$$

For a lattice depth of 10 $E_r$, we calculate a tunneling energy $zJ/E_r \sim 0.12$ and a total scattering rate $\Gamma_{\rm sp} \sim 0.2\,s^{-1}$, giving $T_{\mathcal{V}} \sim 0.6\,$s, in good agreement with the observed time scale. We conclude that on the time scales considered here, photon scattering contributes significantly to the observed heating rate. Although heating due to technical noise is not excluded, the bound above implies that it is not much more severe than photon scattering. We note that in principle, the decay in visibility could be used to measure the heating rate, provided the dependance of the visibility on temperature is known.

When the ramp times is decreased below $\tau_{\rm ramp} = 100\,$ms, the visibility decreases in a qualitatively different way. As shown in Fig. 2b-d, the decay occurs with a much shorter time constant. Note that at long hold times $\tau_{\rm hold} = 800$ ms, the visibility has dropped to $\mathcal{V} \sim 0.2$, a value almost independent of the ramp time. This is consistent with our earlier claim that heating dominates at long times. We attribute the short-times decay to the generation of excitations by a finite ramp speed. To compare the different ramp times, we plot in Fig. 3 the measured visibility as a function of ramp time for a fixed hold time of 300 ms, long enough for the excitations generated during the ramp to relax, but short enough that heating effects do not blur all differences between the different ramps. A time scale of $\sim 100\,$ms clearly emerges, above which the ramp time can be increased without noticeable effect. To check whether this could be an artifact of our plotting method, we have fitted the data to an empirically chosen Lorentzian form. The half-width at half-maximum of the Lorentzian was taken as the visibility decay time. As seen from Fig. 4, the same behavior is found, with a characteristic $1/e$ ramp time $\approx 60\,$ms.

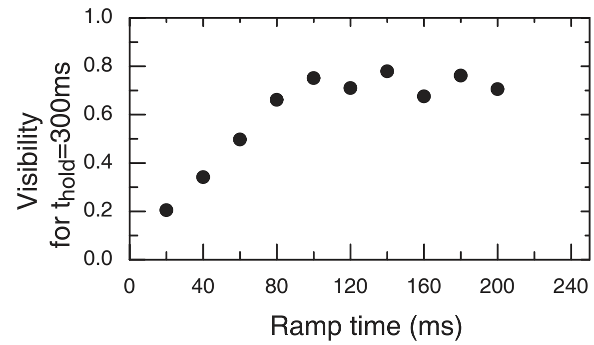


FIG. 3: Measured visibility versus ramp time, for a fixed 300 ms hold time.

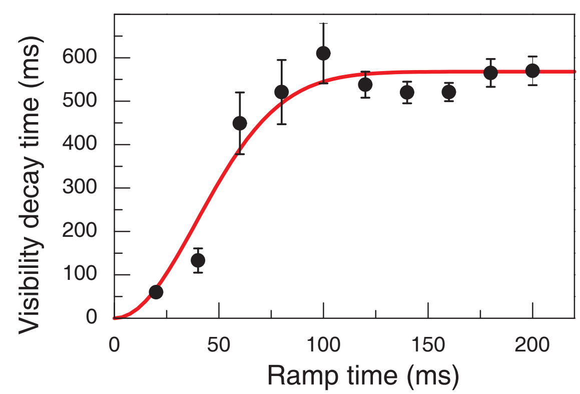


FIG. 4: Measured visibility decay time versus ramp time. The time constant is the half-width at half-maximum of a Lorentzian fit to the data. Error bars indicate the statistical error. The solid line is a fit to an inverted Gaussian, returning a time scale of $\sim 60\,$ms for the visibility decay time to become independent of the ramp.

This time scale can be understood using elementary arguments. Adiabatic evolution in a quantum system with time-dependent hamiltonian requires the condition

$$|\dot{H}| \ll \hbar|\omega_{fi}|^2, \tag{4}$$

where $\dot{H}$ is the derivative of the hamiltonian and where $\omega_{fi}$ is the Bohr frequency of transition between the (instantaneous) eigenstates $|i\rangle$ and $|f\rangle$. Eq. (4) should be fulfilled at all times. For an ultracold gas in an optical

lattice, three energy scales appear: (i) the tunneling matrix element $J$ defining the rate of hopping from site to site, (ii) the on-site interaction energy $U$ and, (iii) the energy associated to the "external" confinement potential usually present on top of the lattice. This potential $V_{\rm ext}$ results from the combination of the magnetic trapping potential in which the condensate is initially produced, and of an additional confinement due to the Gaussian shape of the lasers producing the optical lattice. In practice, $V_{\rm ext}$ is nearly harmonic, with a trapping frequency $\Omega = \omega_{\rm ext} \approx \sqrt{\omega_{\rm m}^2 + 8V_0/mw^2}$, where $\omega_{\rm m} = 2\pi \times 16\,{\rm Hz}$ is the oscillation frequency in the magnetic trap and where $w \approx 136\,\mu$m is the laser beam size. With a proper choice of the laser beam sizes, the increase in $U$ and $\Omega$ are such that the equilibrium size of the condensate as calculated in the Thomas-Fermi approximation varies little during the ramp [20]. In addition, when the lattice depth is increased, $J$ drops exponentially fast whereas $U$ and $\Omega$ increase slowly. Hence, the question of adiabaticity essentially reduces to whether the atoms can redistribute through tunneling in order to adapt the size of the system to the instantaneous Thomas-Fermi shape. This suggests to take $|\dot{H}| \sim |\dot{J}|$ and $\omega_{fi} \sim J/\hbar$ in Eq. (4), giving the following definition for the adiabaticity parameter $\mathcal{A}$,

$$\mathcal{A} = \underset{0\leq t\leq \tau_{\rm ramp}}{\rm Max}\left[\frac{\hbar|\dot{J}|}{J^2}\right]. \qquad (5)$$

For a given ramp, we require $\mathcal{A} \ll 1$ to ensure an adiabatic loading of the cloud into the lattice. In the inset of Fig. 5, we show the quantity $\hbar|\dot{J}|/J^2$ calculated for $\tau_{\rm ramp} = 50$ ms. We have calculated this curve for a final lattice depth of $10E_R$ using the lattice ramp given in Eq. (1), and the analytical estimate [2],

$$\frac{J}{E_r} \approx \sqrt{\frac{8}{\pi}}(V_0/E_r)^{3/4}\exp(-2\sqrt{V_0/E_r}). \qquad (6)$$

The sharp decrease for short times in the inset of Fig. 5 follows from the inadequacy of the tight-binding approximation under which Eq. (6) is derived. We ignore this feature, and calculate $\mathcal{A}$ from the peak value occurring near $t \approx \tau_{\rm ramp}/2$, where the rate of change of the lattice depth is highest. We have repeated the calculation for several ramp times (see Fig. 5). We find that the critical $\mathcal{A} = 1$ corresponds to a ramp time $\tau_{\rm ramp} \approx 80$ ms, close to the experimental findings. This suggests that for our experimental parameters, the loading is indeed limited by single-particle tunneling.

In conclusion, we have studied how the visibility of the interference pattern was affected by the speed at which the lattice was ramped up. A time scale of $\sim 100$ ms was found for adiabatic loading in the optical lattice . In this paper, we focused on the dynamics properties in the superfluid regime. Even more interesting is the dynamical evolution of the system as the superfluid-to-Mott-insulator transition is crossed. An important and still open question is in particular how reversible this transition is. Experiments [1, 16, 17**?** ] found that one could ramp up the lattice intensity and reach the regime where phase coherence is lost, then ramp down the lattice and regain it back. To what extent the initial phase coherent state can be recovered, and what are the limiting mechanisms has however not been studied. Also, the recent availability of virtually exact methods for one-dimensional systems [12] could allow for a precise comparison with the experimental data, in a strongly non-equilibrium regime where theory is still in progression. Finally, new dynamical effects are predicted, such as oscillation of the order parameter and vortices-driven relaxation analog to the Kibble-Zurek mechanism [21].

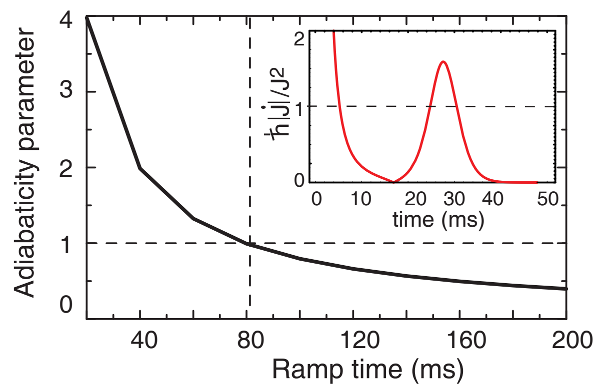


FIG. 5: Adiabaticity parameter (see text) plotted versus ramp time. In the calculation, we have used the actual ramp form as generated in the experiment to describe the lattice depth increase. In the inset, we show how the adiabaticity parameter changes in time as the lattice depth is ramped up. A ramp time of $\tau_{\rm ramp} = 50\,$ms has been used for this plot.

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